adding those products, $2^i q$, which correspond to those quotients of p that are odd numbers. E. g. 51×81. 25, 162; 12, 324; 6, 648; 3, 1,296; 1, 2,592.

$$51 \times 81 = 81 + 162 + 1{,}296 + 2{,}592 = 4{,}131.$$

Prove or disprove the truth of the above proposition.

437. Proposed by C. N. SCHMALL, New York City.

Given that $S_1, S_2, S_3, \cdots, S_k$ are the sums of k arithmetical series, each taken to n terms. The first terms are respectively $1, 2, 3, \cdots, k$, and the common differences are $1, 3, 5, \cdots, (2k-1)$. Show that

$$S_1 + S_2 + S_3 + \cdots + S_k = \frac{nk(nk+1)}{2}.$$

GEOMETRY.

When this issue was made up, no solutions had been received for numbers 446 and 449.

466. Proposed by HORACE OLSON, Chicago, Illinois.

Given the edges of a triangular pyramid, find the radius of the inscribed sphere.

467. Proposed by E. T. BELL, Seattle, Washington.

It is well-known that if i, j, k, l are concyclic points, W_i the Wallace line (frequently, and erroneously, called the Simson line), of i with respect to the triangle jkl, then W_i, W_j, W_k, W_l are concurrent, say in the point $\{i, j, k, l\}$. If $1, 2, 3, \cdots$ denote concyclic points, prove that:

(i) $\{1, 2, 3, 4\}$, $\{1, 2, 3, 5\}$, $\{1, 2, 4, 5\}$, $\{1, 3, 4, 5\}$, $\{2, 3, 4, 5\}$ are concyclic; say on the circle $[1, 2, 3, 4, 5]$;

(ii) Starting with 1, 2, 3, 4, 5, 6, omitting each point in turn, by (i), six circles, are found; these are concurrent, say in the point $\{1, 2, 3, 4, 5, 6\}$;

(iii) Starting with 1, 2, 3, 4, 5, 6, 7, seven points of the kind in (ii) are found; these lie on a circle.

(iv) Continuing thus indefinitely, there is, in each case, finally a unique point or circle according as the number of initial points is even or odd. Also, at any stage, the point of concurrence or the circle bears a simple relation to the initial points: what is it?

[*Note.*—This problem is obviously connected with the theorems of Clifford (*Mathematical Papers*, pp. 38 and 410), but it is interesting to note that it may be completely solved by the methods of Euclid, Books I to III (but at considerable length), and hence is well within the range of high school students. But a very simple proof may be given by the methods of strictly elementary analytical geometry.]

468. Proposed by ELMER SCHUYLER, Brooklyn, New York.

Given two circles and a straight line, to draw a circle tangent to the line and coaxial with the two given circles.

469. Proposed by J. ALEXANDER CLARKE, West Philadelphia High School.

If in an isosceles triangle, a circle is described on one side as diameter, and a line is drawn through the mid-point of the side parallel to the base, the circle and the parallel will intercept on the *tri*sector of the angle at the vertex a segment equal to the radius of the circle. Show how this can be used to trisect any angle.

CALCULUS.

When this issue was made up, no solutions had been received for numbers 336, 338–340, 342, 348, 353, 360, and 363.

387. Proposed by C. N. SCHMALL, New York City.

Show that the volume bounded by the cone $x^2 + y^2 = (a - z)^2$ and the planes $x = 0$, $x = z$, is $\frac{2}{9}a^3$.

388. Proposed by PAUL CAPRON, U. S. Naval Academy.

If $f(x, y) = 0$ represents (in rectangular coordinates) a curve having a simple tangency to the axis of x at the origin, the value of $x^2/2y$, derived from $f(x, y) = 0$, and evaluated for $x = 0$, $y = 0$, will be the radius of curvature at the origin; or if the curve is similarly tangent to the y-axis at the origin, $y^2/2x$, evaluated for $x = 0$, $y = 0$, is the radius of curvature at the origin.

389. Proposed by FRANK R. MORRIS, Glendale, Calif.

A man is at the southeast corner of a section of land and wishes to walk to the opposite corner in the least possible time. A circular track with a radius of $1/\pi$ miles is located in the section tangent to the west line at a point 120 rods from the south line. Conditions are such that he can walk at the rate of 4 miles an hour inside the track and 3 miles an hour outside the track. What course should he choose and how long is it?

MECHANICS.

When this issue was made up, no solutions had been received for numbers 274, 277, 279, 287, 291, and 292.

311. Proposed by B. J. BROWN, Student in Drury College.

A particle oscillates in a straight line about a center of force varying as the distance, and is subject to a retardation $k \times (\text{vel.})^2$. If a, b be two successive elongations, on opposite sides, prove that $(1 + 2ka)e^{-2ka} = (1 - 2kb)e^{2kb}$. What form does the result take if a is infinite? From *Lamb's Dynamics*, p. 299, ex. 14.

312. Proposed by C. N. SCHMALL, New York City.

A ball of elasticity e is projected upward from a point on an inclined plane, so that after its first contact with the plane it rebounds to its starting point. If ϕ be the inclination of the plane to the horizontal, and ψ the angle made by the line of projection with the inclined plane, show that

$$\cot \phi \cot \psi = e + 1.$$

313. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A heavy extensible wire of length c and of constant cross-section w, and density k, is suspended by one end and hangs vertically. If e is the coefficient of elasticity, show that the length of the wire when stretched will be $c(1 + ekgw/2)$.

NUMBER THEORY.

When this issue was made up, no solutions had been received for numbers 188–9, 191–2, 196, 200, 205, 208–9, 211, 214–15, 217, and 219.

232. Proposed by E. T. BELL, Seattle, Washington.

If $F(x)$ is any function of x which vanishes with x, and which, for $0 < |x| \leqq |\xi|$, can be expanded in an absolutely convergent series of positive powers of x, show that a function $f(n)$ may be found, essentially in one way only, such that

$$\int_0^{\xi} \frac{1}{x} F(x)dx = -\log \prod_{n=1}^{\infty} (1 - \xi^n)^{(1/n)f(n)},$$

and find the form of $f(n)$ explicitly in terms of the coefficients in the expansion of $F(x)$. Hence, as particular examples, expand (when possible by this method) e^x as an infinite product, and show that

$$\frac{1}{e} = \prod_{n=1}^{\infty} \left(1 - \frac{1}{2^n}\right)^{(1/n)\phi(n)}$$

where $\varphi(n)$ is the totient of n.

233. Proposed by V. M. SPUNAR, Chicago, Illinois.

Solve in rational numbers $x^2 + y^2 = a^2$, $xy = m/n$, where m and n are integers and relatively prime to each other.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

424. Proposed by S. A. JOFFE, New York City.

Sum the series

$$\binom{n}{a} - \binom{n-1}{a}\binom{i}{1} + \binom{n-2}{a}\binom{i}{2} - \binom{n-3}{a}\binom{i}{3} + \cdots + (-1)^i\binom{n-i}{a}$$

and consider the cases $i = a$ and $i > a$.